

Black Brane Entropy from Brane-Antibrane Systems*

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(Dated: April, 2002)

In the context of string theory, it is possible to explain the microscopic origin of the entropy of certain black holes in terms of D-brane systems. To date, most of the cases studied in detail refer to extremal (supersymmetric) or near-extremal black holes. In this work we propose a microscopic model for certain black branes (extended versions of black holes) which would apply to cases arbitrarily far from extremality, including the Schwarzschild case. The model is based on a system of D-branes and anti-D-branes, and is able to reproduce several properties of the corresponding supergravity solution. In particular, the microscopic entropy agrees with supergravity, except for a factor of $2^{p/p+1}$, where p is the dimension of the brane.

PACS numbers: 11.25.-w, 04.70.Dy, 11.27.+d, 11.10.Wx

I. STRINGS, D-BRANES, AND ENTROPY

String theory replaces point particles with strings, one-dimensional objects whose tension $T_F \equiv 1/2\pi l_s^2$ defines a dimensionful parameter l_s , known as the string length. Besides moving as a whole, a closed string can oscillate in different ways, and upon quantization these internal modes give rise to a perturbative spectrum consisting of an infinite tower of states with masses $m^2 = 4N/l_s^2$, $N = 0, 1, \dots$. At the bottom of the tower there are massless states, corresponding to a graviton $h_{\mu\nu}$, an antisymmetric tensor field $B_{\mu\nu}$, a scalar φ (the dilaton) whose vacuum expectation value determines the string coupling constant $g_s = \exp(-\varphi)$, the so-called Ramond-Ramond (R-R) gauge fields $C_{\mu_1 \dots \mu_{p+1}}$ for various values of p , and the accompanying superpartners. At low energies ($E \ll l_s^{-1}$) these are the only relevant modes, and the effective field-theoretic description is in terms of ten-dimensional Type II supergravity, with Newton's constant $G_N \sim g_s^2 l_s^8$.

The non-perturbative spectrum of string theory contains extended objects of various dimensions, collectively known as branes. Particularly important among these

are the D p -branes [2], solitonic objects extended along p spatial dimensions, whose tension is inversely proportional to g_s . The excitations of a D-brane are described by open strings with endpoints constrained to lie on the brane; quantization of these strings gives rise to another infinite tower of states. At the massless level one obtains a $(p+1)$ -dimensional gauge field A_α , $9-p$ scalars Φ_i describing oscillations of the brane in the transverse directions, and superpartners. An important point is that, in the presence of N parallel D p -branes, there are N^2 different types of open strings (each string can start and end on any one of the branes), which means that the gauge field and scalars become $N \times N$ matrices. At low energies, this system is described by $(p+1)$ -dimensional $U(N)$ super-Yang-Mills theory (SYM) with 16 real supersymmetries and coupling constant $g_{YM}^2 \sim g_s l_s^{p-3}$. If N is large, the strength of the interactions is in effect controlled not by g_{YM}^2 , but by the 't Hooft coupling $g_{YM}^2 N \propto g_s N$.

The other key property of D-branes is that they couple to closed strings: an open string attached to the branes can close, and then wander off into the bulk of ten-dimensional spacetime. This means in particular that D p -branes are sources for the supergravity fields, and so a large collection of them should be describable, at low energies, as a macroscopic solution of supergravity, invariant under translations along p spatial directions. Such solitonic solutions are known as black p -branes [3]; they generically involve a non-trivial metric, dilaton, and R-R gauge field $C_{\mu_1 \dots \mu_{p+1}}$, all expressed in terms of a harmonic function $H(r) = 1 + (R/r)^{7-p}$ in the

*Invited talk by A.G. at the IV Mexican Workshop on Gravitation and Mathematical Physics, based on the original work [1], which should be consulted for a complete reference list.

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$(9-p)$ -dimensional space transverse to the brane, with $R \sim (g_s N)^{1/(7-p)} l_s$ a characteristic length scale. The geometries in question have an asymptotically flat region at large r , which connects at $r \sim R$ to a ‘throat’ extending down to a horizon at $r = 0$.

Supergravity itself is only an approximate description of the low-energy physics of string theory, and so its black brane solutions are reliable only as long as their curvature is small compared to the string scale. This requires $R \gg l_s$, i.e., $g_s N \gg 1$. As we have seen above, perturbative SYM is valid only for $g_s N \ll 1$, so these two alternative descriptions of the D-brane system have mutually exclusive regimes of validity. The supergravity solution corresponding to a stack of N unexcited D-branes is *extremal*: its mass $M = N\tau_p V$ (with V the volume spanned by the brane) and R-R charge $Q = N$ saturate the BPS-type inequality $M \geq |Q|\tau_p V$, which is implied by the supersymmetry algebra for any object carrying $(p+1)$ -form charge Q . BPS saturation is equivalent to the statement that the brane solution preserves some fraction of the supersymmetries (in this case, half).

Black branes are extended versions of black holes— as we have said, all such solutions possess an event horizon. This means in particular that they have an entropy related to their horizon area by the well-known Bekenstein-Hawking formula, $S_{BH} = A_h/4G_N$. Ever since this formula was written down, it has been an outstanding problem to show how this entropy arises from an explicit counting of the microscopic states of the black object. This can in fact be done for the extremal black p -branes discussed above (which upon dimensional reduction may be regarded as black holes in $10-p$ dimensions), but the agreement is trivial: the horizon area of the solutions vanishes, corresponding to the fact that the entropy of their microscopic counterpart, an unexcited N D p -brane system, is zero (there is only one such state).

To attempt to carry out a non-trivial check, we should add some energy to the system (without adding any charge). The black brane is then *non-extremal*, and has a non-vanishing horizon area. The simplest such setup is the *near-extremal* black three-brane, which is understood microscopically as a collection of slightly excited N D3-branes. On the supergravity side, the system has a charge $Q = N$ and a mass $M = N\tau_3 V + \delta M$, where $\tau_3 = 1/(2\pi)^3 g_s l_s^4$ is the D3-brane tension and $\delta M \ll N\tau_3 V$. The Bekenstein-Hawking formula assigns the black three-brane an entropy

$$S_{BH} = 2^{5/4} 3^{-3/4} \sqrt{\pi Q} V^{1/4} \delta M^{3/4} + \dots, \quad (1)$$

where the dots denote corrections which are higher order in δM .

On the microscopic side, the system is understood to be a stack of N D3-branes plus a gas of open strings. The massless open string modes should make the dominant contribution to the entropy, and so we can identify the excess mass with the energy of a gas of massless particles (gluons, scalars, and superpartners): $\delta M = n_b (\pi^2/16) N^2 V T^4$, with T the temperature of the gas and

n_b the number of bosonic (matrix) degrees of freedom (the numerical coefficient then takes their fermionic partners into account). The entropy of the system is entirely due to the gas,

$$S_g = n_b \frac{\pi^2}{12} N^2 V T^3 = n_b^{1/4} \frac{2}{3} \sqrt{\pi Q} V^{1/4} \delta M^{3/4}. \quad (2)$$

The functional dependence of S_g and S_{BH} on δM and Q is identical; the numerical coefficient agrees if $n_b = 6$ [4]. This number should be contrasted with the $g_s N \ll 1$ result, $n_b = 8$, arising from the two transverse polarizations of the gauge field A_α and the six scalars Φ_i . Since the supergravity result applies only in the $g_s N \gg 1$ regime, this discrepancy is not a contradiction. To date, the best understanding of this case comes from the AdS/CFT correspondence [5]: taking an appropriate low-energy limit reduces the black three-brane solution to $\text{AdS}_5 \times \mathbf{S}^5$, and at the same time makes $(3+1)$ -dimensional $SU(N)$ SYM an exact description of the system of D-branes and open strings. The exact equivalence thus obtained between string theory on the ten-dimensional anti-de Sitter background and the four-dimensional gauge theory then *predicts* in particular that the SYM entropy at $g_{YM}^2 \ll 1$, $g_{YM}^2 N \gg 1$ is the same as that of a free field theory with only $6N^2$ bosons and the same number of fermions, instead of the naive $8N^2$.

The first successful account of black brane entropy in terms of the open strings living on D-branes was obtained by Strominger and Vafa [6], who were able to reproduce the exact entropy of certain extremal (yet finite-horizon-area) black holes in five dimensions through microscopic state counting in a D5-brane/D1-brane system. This was followed by a tremendous surge of activity [7], which led to the discovery of the AdS/CFT correspondence [5] as a remarkable by-product, and continues even today.

Most of the examples where a detailed microscopic description has been found involve extremal and near-extremal black branes. These systems have positive specific heat, and their entropy can be accounted for as in (2), using a gas of finite temperature living on the D-branes. Black branes which are far from extremality, like the ordinary Schwarzschild black hole, are in this respect much more challenging. Consider for example the case of a *neutral* black three-brane of mass M . Its entropy is given by [7]

$$S_{BH} = 2^{\frac{9}{4}} 5^{-\frac{5}{4}} \pi^{\frac{1}{4}} \sqrt{\kappa} M^{\frac{5}{4}} V^{-\frac{1}{4}}, \quad (3)$$

which we have expressed in terms of the gravitational coupling $\kappa \equiv \sqrt{G_N/8\pi}$. If one tries to interpret this as the entropy of a gas of particles with energy $E = M$ some well-known problems arise. First, since the power of E is larger than one, a simple thermodynamical calculation shows that the specific heat is negative. Second, by extensivity the exponent of V is forced to be negative, which implies that the pressure $p = T(\partial S/\partial V)_M$ is negative. Finally, when the black brane evaporates completely through Hawking radiation, all of its excitations

disappear, meaning that the field theory which describes it microscopically should have a vacuum with no degrees of freedom (other than closed strings).

As we will see, these same properties are in fact possessed by D-brane–anti-D-brane systems, which have been much studied of late [8]. An anti-D p -brane ($\overline{\text{D}p}$ -brane) is simply a D p -brane with reversed orientation—it has the same mass $M = N\tau_p V$ as the D-brane but opposite charge, $Q = -1$. A system with N D3-branes and the same number of $\overline{\text{D}3}$ -branes is therefore a natural candidate to describe the neutral black three-brane [1]. We will study this system in the next section.

Other string-theoretic attempts to give a microscopic description of Schwarzschild black holes have been made previously, most notably via the string/black hole correspondence [9], and in the Matrix theory [10] context [11].

II. BRANE-ANTIBRANE SYSTEMS AT FINITE TEMPERATURE

The excitations of a D3– $\overline{\text{D}3}$ pair are described by open strings whose endpoints are anchored on the branes. The 3–3 and $\overline{3}$ – $\overline{3}$ strings give rise to the usual massless gauge fields and scalars $\{A_\alpha, \Phi_i\}$, $\{\overline{A}_\alpha, \overline{\Phi}_i\}$ (plus superpartners) on the brane and the antibrane. The 3– $\overline{3}$ and $\overline{3}$ –3 strings, on the other hand, yield a complex scalar field ϕ with negative mass-squared (i.e., a tachyon), and additional massless fermions. Since it originates from strings running between the brane and the antibrane, the tachyon is charged under the relative $U(1)$ (i.e., $A_\mu^- \equiv A_\mu - \overline{A}_\mu$), but is neutral under the overall $U(1)$ ($A_\mu^+ \equiv A_\mu + \overline{A}_\mu$).

The tachyonic mass of ϕ expresses an instability of the system, the meaning of which becomes clear upon examination of the tachyon potential [12],

$$V(\phi) = 2\tau_3 \exp[-2|t(\phi)|^2], \quad (4)$$

with t related to ϕ through an error function,

$$|\phi| = \sqrt{\frac{\pi}{2}} \text{Erf}(|t|). \quad (5)$$

The potential (4) is of the ‘Mexican hat’ type: it has a maximum $V = 2\tau_3$ at $\phi = 0$ (the unstable ‘open string’ vacuum) and a minimum $V = 0$ at $|\phi| = \sqrt{\pi/2}$ (the ‘closed string’ vacuum). From the energy difference between the two vacua we deduce that condensation of the tachyon from the former to the latter corresponds to the annihilation of the brane and the antibrane, which in particular implies the disappearance of all open string degrees of freedom [13].

If we now consider this same system at a finite temperature, standard thermal field theory reasoning tells us what to expect: a small temperature should lead to an effective potential in which the location of the minimum has shifted away from $|\phi| = \sqrt{\pi/2}$. The physical reason

for this is that moving towards $\phi = 0$ can be thermodynamically favorable: it costs energy, but it also reduces the mass of the tachyon and therefore increases the entropy of the tachyon gas. The optimal configuration is the one that minimizes the free energy of the system, and this will vary with the temperature.

Let us now determine whether and under what conditions the open string vacuum, $t = \phi = 0$, can be a minimum of the effective potential. When this happens, the ‘tachyon’ will no longer be tachyonic, and so (at sub-string-scale temperatures) the massless fields will make the most important contribution to the free energy. If one starts at $\phi = 0$, then sliding down the tachyon potential (4) lowers the energy of the system, but it also gives mass to the relative gauge fields, and so decreases the entropy of the gas. We are interested in establishing which of these effects dominates.

Our system consists of N brane-antibrane pairs, a tachyon condensate, and a gluon (+ transverse scalars + superpartners) gas. Its free energy at temperature $T = \beta^{-1}$ is given by

$$F(\phi, \beta) = 2\tau_3 \text{Tr} e^{-2|t|^2} + \frac{4\pi}{(2\pi)^3} c N^2 \beta^{-4} \quad (6)$$

$$\times \int_0^\infty dx x^2 \ln \left[\frac{1 - e^{-\sqrt{x^2 + \beta^2 m^2}}}{1 + e^{-\sqrt{x^2 + \beta^2 m^2}}} \right],$$

where t is an $N \times N$ matrix, $m \sim |\phi|$ is the mass given to the gluons by the Higgs effect, and the numerical constant $c = 8$ for the relevant 8 bosonic + 8 fermionic degrees of freedom. Starting at $\phi = 0$ and letting a single diagonal tachyon condense by an amount $\delta\phi$ gives mass to N out of the N^2 species of particles in the gas, and so changes (6) by

$$\delta F = -4\tau_3 (\delta\phi)^2 + \frac{1}{2} N \beta^{-2} (\delta m)^2, \quad (7)$$

which is positive for large enough temperature. Disregarding the numerical constants, we thus learn that for

$$T \geq \frac{1}{\sqrt{g_s N} l_s}. \quad (8)$$

the open string vacuum is a minimum of the free energy (equivalently, a maximum of the entropy for fixed total energy), and the brane-antibrane pairs do not annihilate.

To arrive at (7) we have considered the mass that the relative gauge fields (and scalars) acquire due to their coupling to the tachyon, but we can equivalently phrase the result in the opposite direction: the second term of (7) represents a mass term for ϕ due to a thermal expectation value for the relative gauge fields, $\langle A^- A^- \rangle_T \sim NT^2$, corresponding to a mass $m_\phi \sim \sqrt{g_s N} T$.

It is important to note that the regime (8) lies in the physically accessible sub-string-scale region only for non-perturbative values of the coupling, $g_s N > 1$, where we would expect the system to have a dual supergravity description. The system under consideration becomes then a viable candidate for the desired microscopic description of a neutral black three-brane.

III. MICROSCOPIC MODEL

Given the considerations of the previous sections, the model that we consider [1] is the low-energy theory on the worldvolume of a system of N D3-branes and N anti-D3-branes. Since the number of uncondensed branes and antibranes can vary, the actual values are chosen to maximize the total entropy of the system for fixed charge and mass.

The temperature is assumed (and later confirmed) to satisfy $\frac{1}{\sqrt{g_s N}} \ll T \ll 1$ in string units. As seen in (8), the first inequality is required for stability; the second one allows us to ignore the massive open string modes. For the above temperature range to exist we must have $g_s N \gg 1$, so we are necessarily in the strong-coupling regime. We will take $g_s \ll 1$ to suppress closed string loops. Under these conditions, when a brane-antibrane pair annihilates the energy goes to the gas of open strings on the remaining branes and antibranes, rather than being emitted as closed strings (Hawking radiation), since the latter process is disfavored for small g_s .

Since we are trying to formulate a microscopic model for a supergravity solution, the restriction to strong coupling was of course expected. In the absence of a weakly-coupled regime, and knowing that the theory is not supersymmetric, the best one can do is to use plausibility arguments to determine the entropy. From the structure of the brane-antibrane worldvolume theory one can show that, if $\mathcal{O}(N^2)$ fields are excited on the branes and antibranes, then the tachyons and the fermions acquire a mass of order $\sqrt{g_s N} T$, which is in fact what we obtained (for the tachyon) in Section II. If such is the case, the fact that (for $g_s N \gg 1$) the temperature is much lower than their mass means that these fields are not excited, and the theory on the branes decouples from the theory on the antibranes. We then have two copies of $(3+1)$ -dimensional $\mathcal{N} = 4$ $U(N)$ SYM. (The supersymmetries preserved by each copy are different, so the overall system is not supersymmetric.) As we have seen in Section I, at strong-coupling the AdS/CFT correspondence [5] predicts that the entropy of each copy is the same as that of a free field theory with $6N^2$ bosons and the same number of fermions [4].

Given this set of assumptions, the energy and entropy of our microscopic system follow as

$$M_{\text{FT}} = 2N\tau_3 V + n_b \frac{\pi^2}{8} N^2 V T^4, \quad (9)$$

$$S_{\text{FT}} = n_b \frac{\pi^2}{6} N^2 V T^3, \quad (10)$$

where $n_b = 6$ is the number of bosonic degrees of freedom. Notice that, while the brane/antibrane tension and the energy of the two gases contribute with the same sign to M_{FT} , and therefore to the time-time component of the energy-momentum tensor, $T_{00} = \mathcal{E}_D + \mathcal{E}_g$, they contribute with opposite signs to the pressure: $T_{ij} = (-\mathcal{E}_D + \frac{1}{3}\mathcal{E}_g)\delta_{ij}$ (remember that $\mathcal{E}_D \equiv 2N\tau_3$ arises from the potential of a scalar field).

From (9) and (10) we obtain

$$S_{\text{FT}} = n_b^{\frac{1}{4}} \frac{2^{\frac{5}{4}}}{3} \sqrt{\pi} \sqrt{N} V^{\frac{1}{4}} (M_{\text{FT}} - 2NV\tau_3)^{\frac{3}{4}}, \quad (11)$$

which is maximized by $N = M_{\text{FT}}/5\tau_3 V$. This is easily seen to imply that the energy contained in the gases is $3/2$ of the total tension of the branes and antibranes, a prediction which will be shown below to agree with supergravity. In addition, we find that the temperature is $T \sim (g_s N)^{-\frac{1}{4}}$, which as required satisfies $\frac{1}{\sqrt{g_s N}} \ll T \ll 1$. Plugging the equilibrium value of N back into (11) we obtain the entropy-energy relation

$$S_{\text{FT}} = n_b^{\frac{1}{4}} 2^{\frac{5}{4}} 3^{-\frac{1}{4}} 5^{-\frac{5}{4}} \pi^{\frac{1}{4}} \sqrt{\kappa} V^{-\frac{1}{4}} M_{\text{FT}}^{\frac{5}{4}}, \quad (12)$$

where we have expressed the D3-brane tension in terms of the gravitational coupling constant, $\tau_3 = \sqrt{\pi}/\kappa$.

To compare with supergravity, we recall that a neutral black three-brane with Schwarzschild radius r_0 has mass and entropy [7]

$$M_{\text{SG}} = \frac{5}{2} \frac{\pi^3}{\kappa^2} r_0^4 V, \quad (13)$$

$$S_{\text{SG}} = \frac{2\pi^4}{\kappa^2} r_0^5 V, \quad (14)$$

which implies that

$$S_{\text{SG}} = 2^{\frac{9}{4}} 5^{-\frac{5}{4}} \pi^{\frac{1}{4}} \sqrt{\kappa} V^{-\frac{1}{4}} M_{\text{SG}}^{\frac{5}{4}}. \quad (15)$$

Identifying $M_{\text{SG}} = M_{\text{FT}}$, we see that the functional form of the supergravity and field theory entropies agree. With $n_b = 6$, the numerical coefficient does not quite agree: the field theory entropy is a factor of $2^{3/4}$ too small, $S_{\text{SG}} = 2^{3/4} S_{\text{FT}}$. Equivalently, one can say that the supergravity entropy behaves as if the gases carried twice the available energy.

Another interesting check is to consider the energy-momentum tensor. From the asymptotic value of the gravitational field one finds that [14] $T_{ij} = -\frac{1}{5} T_{00} \delta_{ij}$. Putting $T_{00} = \mathcal{E}_D + \mathcal{E}_g$ and $T_{ij} = (-\mathcal{E}_D + \frac{1}{3}\mathcal{E}_g)\delta_{ij}$, as discussed above, one obtains $\mathcal{E}_g = \frac{3}{2}\mathcal{E}_D$, in agreement with the field theory prediction.

Before moving on to the charged case, let us discuss an interesting issue that appears already here. Since we reproduce the black brane entropy, it is clear that the specific heat of our system is negative. To understand why, let us consider how Hawking evaporation proceeds in this model, and check that the temperature increases when energy is radiated. When a closed string is emitted, energy is taken from the open string gas, so a priori the temperature should decrease. However, we have found that, in equilibrium, the energy in the gas is $3/2$ the tension of the $D-\bar{D}$ pairs. This means that, when the gas has lost enough energy so as to match $3/2$ the tension of $N-1$ pairs, it will be entropically favorable for one pair to annihilate, giving energy to the gas and increasing its temperature. This is repeated again and

again, effectively increasing the temperature on average as the mass of the system decreases. The process will continue until $g_s N \sim 1$, where the gas temperature becomes of order one in string units, and the model (16)-(18), based only on the massless open string modes, ceases to be valid. At this point we would expect all of the available energy to go into a highly-excited long string, so the brane-antibrane model makes contact with the string/black hole correspondence [9].

It is straightforward to generalize the model to the case of a charged black brane, where the numbers of D3-branes and anti-D3-branes are no longer equal, $N \neq \bar{N}$:

$$M_{\text{FT}} = (N + \bar{N})\tau_3 V + E_g + E_{\bar{g}}, \quad (16)$$

$$S_{\text{FT}} = 2^{\frac{5}{4}} 3^{-\frac{3}{4}} \pi^{\frac{1}{2}} V^{\frac{1}{4}} \left(E_g \sqrt{N} + E_{\bar{g}} \sqrt{\bar{N}} \right), \quad (17)$$

$$Q_{\text{FT}} = N - \bar{N}, \quad (18)$$

$$T_{\text{FT}}^{ii} = -(N + \bar{N})\tau_3 + \frac{1}{3V} (E_g + E_{\bar{g}}), \quad (19)$$

where $E_g = 3\pi^2 N^2 V T^4 / 8$ ($E_{\bar{g}} = 3\pi^2 \bar{N}^2 V \bar{T}^4 / 8$) is the energy of the gas on the branes (antibranes). Notice that, since the theories are decoupled, the two gases could a priori have different temperatures, $T \neq \bar{T}$.

Let us now consider the supergravity expressions, following a procedure similar to [15]. The energy, entropy and charge of a non-extremal three-brane are given by

$$M_{\text{SG}} = \frac{\pi^3}{\kappa^2} r_0^4 V \left(\cosh 2\alpha + \frac{3}{2} \right), \quad (20)$$

$$S_{\text{SG}} = \frac{2\pi^4}{\kappa^2} r_0^5 V \cosh \alpha, \quad (21)$$

$$Q_{\text{SG}} = \frac{\pi^{\frac{5}{2}}}{\kappa} r_0^4 \sinh 2\alpha. \quad (22)$$

As discussed before, it is interesting to consider not only the mass but also the other components of the energy-momentum tensor, which turn out to be

$$T_{\text{SG}}^{ij} = \left[\frac{\pi^3}{\kappa^2} r_0^4 \left(-\cosh 2\alpha + \frac{1}{2} \right) \right] \delta^{ij}. \quad (23)$$

Comparing (20), (22) and (23) with (16), (18) and (19) uniquely determines

$$N = \frac{\pi^{\frac{5}{2}}}{2\kappa} r_0^4 e^{2\alpha}, \quad \bar{N} = \frac{\pi^{\frac{5}{2}}}{2\kappa} r_0^4 e^{-2\alpha}, \quad (24)$$

and identifies the energy $E \equiv E_g + E_{\bar{g}}$ of two the gases as

$$E = \frac{3}{2} \frac{\pi^3}{\kappa^2} V r_0^4, \quad (25)$$

in terms of which the entropy can be written as

$$S_{\text{SG}} = 2^{\frac{5}{4}} 3^{-\frac{3}{4}} \pi^{\frac{1}{2}} V^{\frac{1}{4}} E^{\frac{3}{4}} \left(\sqrt{N} + \sqrt{\bar{N}} \right). \quad (26)$$

From (17), we see that this is the correct entropy for a gas of particles on the N branes plus another gas on the \bar{N}

antibranes, both with the same energy E . However, since the total energy available for the gases is E , in the field theory model we have to assign an energy $E/2$ to each of them, resulting in a mismatch in the entropy which is exactly the same as found for the neutral case: $S_{\text{SG}} = 2^{3/4} S_{\text{FT}}$. Under the condition that the energies in both gases are the same, one can check that the expressions (24) are the ones that maximize the entropy for fixed charge and mass.

The fact that the energy densities of the two gases are the same implies that their temperatures T, \bar{T} are different. They are related through

$$\frac{2}{T_{\text{FT}}} = \frac{1}{T} + \frac{1}{\bar{T}}, \quad (27)$$

where T_{FT} is the temperature defined as $T_{\text{FT}}^{-1} = (\partial S / \partial M)_Q$ (which is a factor of $2^{3/4}$ smaller than the Hawking temperature, $T_H = 1/\pi r_o \cosh \alpha$). It would be interesting to try to detect these two distinct temperatures directly in the supergravity side, as was done for the D1/D5 system in [16].

As we emphasized before, there is nothing to prevent us from postulating that the temperatures of the two gases are different, but it is not clear to us why supergravity seems to require that their energies (or equivalently, the pressures,) be the same. The equality of the two energies implies that, as one approaches the extremal limit $M = Q\tau_3 V$ (i.e., $\alpha \rightarrow \infty$ with $r_0 \propto e^{-\alpha/2}$), the temperature of the gas on the antibranes grows without bounds. Since the model is based on massless open string modes, it is expected to be valid only if both T and \bar{T} are substantially lower than the string scale. We expect that as $\bar{T} \rightarrow l_s^{-1}$, the energy available to the gas on the antibranes goes to a highly excited long string, whose contribution to the entropy is however negligible compared to the gas on the branes.

As reviewed in Section I, in the near-extremal region it is known that the black brane entropy can be precisely reproduced using a system without antibranes [4], so as one approaches extremality one would intuitively expect a transition to this class of states. It is easy to see that the entropies of these two microscopic descriptions cross when $M \simeq 6Q\tau_3 V$, which indeed suggests a transition, with the brane-antibrane system being the preferred one further away from extremality. However, it is hard to see how one could retain the agreement with supergravity in the entire parameter space: the brane-antibrane model gives the exact dependence of the black brane entropy on M and Q , but gives a numerical value which is always a factor of $2^{3/4}$ too small, whereas the model of [4] is in accord with supergravity in the near-extremal region, but deviates significantly from it already at the presumed transition point.

Another important property of black branes is that they are generically unstable [17]. If the black three-brane, for instance, lives on a very large torus, then it has lower entropy than a ten-dimensional black hole, so the latter is the preferred configuration. It can be shown

[1] that in our microscopic model it is also convenient to reduce the size of the system, but only until it is of the order of the inverse temperature. The radius that maximizes the entropy is found to yield an entropy-energy relation which is precisely that of a black hole in ten dimensions, as expected from the supergravity side.

Before closing, we should note that entirely analogous models can be formulated [1] for the black two-brane and five-brane of eleven dimensional supergravity, where the microscopic description is based upon $M2-\overline{M2}$ and $M5-\overline{M5}$ systems in M-theory. Using again the input from AdS/CFT, the results are in complete parallel with those of the $D3-\overline{D3}$ case.

IV. CONCLUSIONS

We have shown that a $D-\overline{D}$ system is stable if the temperature satisfies $T > 1/\sqrt{g_s N} l_s$ (which in turn requires $g_s N > 1$). Building upon this observation and previous work [15], we have formulated an explicit microscopic model for the black three-brane of ten-dimensional type IIB string theory. Combined with input from the AdS/CFT correspondence, the model yields an entropy which reproduces the supergravity result, up to a puzzling factor of $2^{3/4}$. Since the AdS/CFT correspon-

dence uses supergravity (in the near-extremal region), the agreement might appear to be merely a consistency check, but the fact is that we are describing a very different regime (including the Schwarzschild case), and a different situation, since the number of branes can change. As we have seen, our field theory model possesses several appealing features which are in close correspondence with the properties of black branes and black holes in supergravity. Beyond the immediate task of questioning its assumptions and attempting to resolve the numerical discrepancy in the entropy, we believe that there are several interesting aspects of the model which merit further study.

V. ACKNOWLEDGEMENTS

AG would like to thank the organizers of the IV Mexican Workshop on Gravitation and Mathematical Physics for the invitation to present this work. UD is a Royal Swedish Academy of Sciences Research Fellow supported by a grant from the Knut and Alice Wallenberg Foundation and by the Swedish Natural Science Research Council (NFR). The work of AG is supported by a repatriation grant from the Mexican National Council of Science and Technology (CONACyT).

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